J. Math. Soc. Japan
Vol. 18, No. 2, 1966

The structure of K_Λ-rings of the lens space and their applications

By Tsunekazu KAMBE

(Received June 14, 1965)

Introduction.

In [2] M. F. Atiyah used the Grothendieck ring $KO(M)$, of real vector bundles over a differentiable manifold M, to the problems of immersion and imbedding of M, and applied his methods to the n-dimensional real projective space RP^n whose $KO(RP^n)$ had been determined by J. F. Adams [1].

In this paper we shall consider the lens space $L^n(p)$ which is defined as follows: Let p be an integer >1 and γ be the rotation of $(2n+1)$-sphere

$$S^{2n+1}=[(z_0, z_1, \cdots, z_n)/\sum_{i=0}^{n}|z_i|^2=1]$$

of the complex $(n+1)$-space C^{n+1} given by

$$\gamma(z_0, z_1, \cdots, z_n)=(e^{2\pi i/p}z_0, e^{2\pi i/p}z_1, \cdots, e^{2\pi i/p}z_n).$$

Then γ generates the topological transformation group Γ of S^{2n+1} of order p, and the lens space is defined to be the orbit space:

$$L^n(p)=S^{2n+1}/\Gamma.$$

This is the compact differentiable $(2n+1)$-manifold without boundary and in particular $L^n(2)=RP^{2n+1}$.

The reduced Grothendieck rings $\tilde{K}(L^n(p))$ (for prime p) and $\widetilde{KO}(L^n(p))$ (for odd prime p), of complex and real vector bundles over $L^n(p)$ respectively, are determined by the following two theorems.

Let η be the canonical complex line bundle over the complex projective space CP^n. Consider the natural projection

$$\pi: L^n(p)=S^{2n+1}/\Gamma \to S^{2n+1}/S^1=CP^n$$

and the element

$$\sigma=\pi^!(\eta-l_c)^{1)} \in \tilde{K}(L^n(p))$$

1) Throughout this paper, the trivial real (complex) bundle of dimension n will be simply denoted by n (n_c).