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Note on regular sequence spaces

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Let μ be a normal sequence space (see [2] for all definitions). We say that μ is *regular* ([1]) if there exists a perfect space $\lambda \supset \mu$ such that μ is the $\mathfrak{T}_b(\lambda^\times, \lambda)$-closure of φ, the set of finitely non-zero sequences. We say that μ satisfies condition (A) ([2]) if for every sequence (N_n) of pairwise disjoint non-empty finite sets of positive integers and every $x=(x_i) \in \mu$, there exists $y \in \mu$ and a subsequence (N_{n_ν}) of (N_n) such that

$$\lim_{\nu\to\infty} \inf_{i\in N_{n_\nu}} \left| \frac{y_i}{x_i} \right| = \infty .$$

T. Kōmura and Y. Kōmura prove ([1]) that if μ satisfies condition (A) then it is regular and leave the converse as an open question. It is the purpose of this note to show that the converse is false, even if $\lambda=\mu$.

It will be convenient to think of a sequence as a matrix with countably many rows and countably many columns. Let $a^{(k)}$ be the matrix whose first $k-1$ rows each have j^k as the number in the j^{th} column and whose $(k+\nu)^{th}$ row $(\nu=0, 1, \cdots)$ has $k^{k+\nu}$ in every column. Let μ be the set of matrices whose terms are dominated by some $a^{(k)}$, $k=1, 2, 3, \cdots$. This, of course, is the "Stufenräume" of Köthe ([2]) with the steps $a^{(k)}$ and this particular example is, in fact, discussed by him ([2], p. 436). We obtain from [2], § 30, 9, (1) that $\mu[\mathfrak{T}_k(\mu^\times, \mu)]$ is a Montel space and hence barreled, so $\mathfrak{T}_k(\mu^\times, \mu)=\mathfrak{T}_b(\mu^\times, \mu)$ and from [2], § 30, 7, (1), it follows that φ is $\mathfrak{T}_b(\mu^\times, \mu)$-dense in μ. That is, μ is regular with $\lambda=\mu$. Now let $x=a^1 \in \mu$ and let

$$N_n=\{(i, n)/i \leqq n\} \cup \{(n, j)/j \leqq n\} .$$

Clearly, as soon as $n \geqq k$, N_n intersects the k^{th} row of the matrix. Further, if $y \in \mu$, there exists k such that for some $c>0$, $|y_{ij}| \leqq c a_{ij}^k$. Hence, since $x_{ij}=1$, we have,

$$\lim_{\nu\to\infty} \inf_{i,j\in N_{n_\nu}} \left| \frac{y_{ij}}{x_{ij}} \right| \leqq c \lim_{\nu\to\infty} \inf_{i,j\in N_{n_\nu}} a_{ij}^k \leqq c \lim_{\nu\to\infty} k^k = ck^k < \infty .$$

Hence condition (A) is violated.

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