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Abelian varieties attached to automorphic forms

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Introduction.

Let G be a discontinuous group acting on the upper half-plane $\mathfrak{X}$. As a subgroup of $GL(2, \boldsymbol{R})$, G admits a tensor representation M_n of degree n. One can then define the cohomology groups $H^1(M_n, G)$ after Eichler [**1**], and from Shimura [**6**], there exists a canonical isomorphism between $H^1(M_n, G)$ and the space $S_{n+2}(G)$ of cusp forms of degree $n+2$ with respect to G. Under certain "integrality" assumptions on G (for example, when $G=SL(2, \boldsymbol{Z})$, these conditions are satisfied), he defines a lattice in $H^1(M_n, G)$ and proves that the torus so obtained, admits a canonical structure of an abelian variety.

Suppose more generally, we have two discontinuous groups $G \subset G_1$ (G normal in G_1 and $(G_1 : G) < \infty$). Then, associated with a real representation R of G_1/G, we can define the cohomology groups $H^1(R \otimes M_n, G_1)$ and establish a canonical isomorphism between $H^1(R \otimes M_n, G_1)$ and the space $S_{n+2,R}(G_1)$ of vectors of cusp forms of degree $n+2$ with respect to G which remains invariant under the representation R (cf. Theorem 1). If then R is rational and G_1 satisfies the "integrality" assumption [**6**], a lattice in $H^1(R \otimes M_n, G_1)$ can be defined, and as in the case of Shimura, this torus can be endowed with a canonical structure of an abelian variety (say) $A_{n+2,R}(G_1)$. In the special case $G_1 = \Gamma(1)$, $G=\Gamma_1(q)$ (q, a prime) and $n=0$, these have been noticed by Hecke [**4**].

We note finally that these abelian varieties provide a decomposition of $A_{n+2}(H)$ for any subgroup H with $G \subset H \subset G_1$. Further in the special case $G_1 = \Gamma(1)$, $G=\Gamma_1(q)$, one can define Hecke operators τ_r (for r prime to q) as endomorphisms of these abelian varieties.

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It was noticed by the author, after the preparation of the manuscript that Gunning has also proved Theorem 1 in [**2**], but however our proof is different.

NOTATIONS.

$\Gamma(1) = SL(2, \boldsymbol{Z}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \text{ with } a, b, c, d \text{ integral and } ad-bc=1 \right\}$