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On flat modules over commutative rings

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It is well-known that if R is a Noetherian ring or a local ring, then every finitely generated flat R-module is projective (cf. (1), (3), R need not be commutative.). It is also known that if R is a commutative integral domain, the same conclusion holds (cf. (5) Appendix).

In the present paper, a fairly general sufficient condition for a commutative ring R to the effect that the same conclusion holds, will be given as Theorem 2. It will include all the mentioned results as far as they concern commutative rings. This will be deduced from a more general result, Theorem 1, which is obtained by a homological method as used in (4). We shall add another proof of Theorem 2, which is independent from homological method. Finally we shall examine if the converse of Theorem 2 is true. We could not decide this problem, but proved that this is true in some special cases.

Throughout this paper a ring means a commutative ring with unit element. A local ring means a ring with only one maximal ideal and a semi-local ring means a ring with a finite number of maximal ideals.

Let R be a ring, M be an R-module and S be a multiplicatively closed subset of R. Then the quotient ring and the quotient module of R, M with respect to S are denoted by R_S, M_S, respectively. If S is the complementary set of a prime ideal $\mathfrak{p}$ in R, then we shall use $R_{\mathfrak{p}}$, $M_{\mathfrak{p}}$ instead of R_S, M_S. We shall denote by T the set of all non-zero divisors of R. Then the quotient ring R_T of R with respect to T will be called the total quotient ring of R and denoted by K.

An R-module M is called a torsion-free module if, whenever $tu=0$, $u \in M$, $t \in T$, we have $u=0$. On the other hand an R-module M is called a divisible module if for any $t \in T$, $u \in M$ there is an element v of M with $u=tv$.

The other notations and terminologies are the same as in (1).

1. We begin with

LEMMA 1. *Let R be a ring with the total quotient ring K and M be a finitely generated torsion-free R-module such that M_T is K-projective. Then there exists a finitely generated free R-module F such that $F \supset M$ and $(F/M)_T$ is K-projective.*