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Mappings defined on 0-dimensional spaces and dimension theory

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§1. Introduction.

The following is a well-known Hurewicz-Kuratowski's theorem for separable metric spaces R and A (W. Hurewicz [**4**], C. Kuratowski [**6**; **7**]):

In order that a non-empty space R has the covering dimension $\leqq n$, it is necessary and sufficient that there exist a space A with $\dim A = 0$ *and a closed continuous mapping f of A onto R such that the order of f is at most $n+1$.*

In the above $\dim A$ denotes the covering dimension of A, and the order of f is the supremum of $\{|f^{-1}(x)|\ ;\ x \in R\}$, where $|f^{-1}(x)|$ are the cardinal numbers of the sets $f^{-1}(x)$. This theorem has been extended by K. Morita [**14**] to the case when R and A are metric spaces. The classical Hurewicz-Kuratowski's theorem had been rather isolated from the general trends of dimension theory for separable metric spaces. In the framework of dimension theory for general metric spaces which has been constructed by the author this theorem occupies an important position [**17**, § 3]. It seems to the author that closed mappings defined on 0-dimensional spaces will be one of powerful instruments to clear up the relation between the covering dimension and the inductive one of non-separable spaces.

In §§ 2 and 3 we shall characterize a non-metrizable space R which has the following property:

(*) *R is the image of a 0-dimensional space under a closed continuous mapping of order $\leqq n+1$.*

It will be shown that a space has this property if and only if there exists a directed family of closed coverings of order $\leqq n+1$, which follows out the topology of a space (cf. Definitions 2.1 and 2.2 below). We shall notice in § 4 that the inductive dimension of a space which admits a directed family with the property stated above cannot be greater than n. It is to be noted that Theorem 4.1 below has been obtained independently by Soviet mathematicians, I. Proskuryakov—B. Ponomarev—B. Pasynkov, under a more restrictive assumption (P. Alexandroff [**1**, p. 80], B. Pasynkov [**21**]). It is also to be noted that Corollaries 4.2 and 4.4 had been essentially proved by K. Morita (cf. Remark 4.7). As an immediate consequence of our results it will be shown, with the