

A formal system of first-order predicate calculus with infinitely long expressions

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The first-order predicate calculus with infinitely long expressions is being developed by Berkeley school. We shall give a formal system for this calculus and prove a "Main Theorem" which implies the completeness theorem in Gödel's sense, and the cut-elimination theorem, Löwenheim-Skolem theorem and Craig's interpolation theorem.

Our formal system will be obtained from Gentzen's calculus $LK^{(1)}$ by extending the concepts 'formula' and 'sequent', and by moderating the restriction on eigenvariables. This moderation of the restriction on eigenvariables plays an important rôle in making our system complete. We shall first explain by way of introduction, our moderation of the restriction on eigenvariables in case of LK .

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§1. A moderation on the restriction on eigenvariables in the ordinary predicate calculus.

In the Gentzen calculus LK , the restriction on eigenvariables is stated as follows:

The eigenvariable of an inference 'introduction of \forall in succedent' or 'introduction of \exists in antecedent' shall not occur in its conclusion.

Now we shall consider the new calculus obtained from LK by replacing the restriction on eigenvariables by the following one:

Every formal proof satisfies the following three conditions:

- 1) The principal formulae of inferences which have a same eigenvariable are all one and the same formula.
- 2) For each free variable, a non-negative integer named the *height* can be

1) Cf. Gentzen [2]. English terminologies on LK will be used, for most part, according to the usage in Kleene's text book [6].