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On L-series of normal varieties

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Introduction.

Let U and V be normal varieties defined over a finite field k with q elements, and assume that U is a Galois covering of V with the Galois group $\mathfrak{G}$. Under these circumstances several authors defined the L-series associated with the characters of $\mathfrak{G}$. In [**7**], Lang introduced an L-series following the original idea of Artin [**2**] and proved the density theorem. But in his definition the singular points and the branch points of V are all neglected. For his purposes it is sufficient, but for other purposes it may be inconvenient. We shall give, borrowing the ideas in [**3**], [**4**], a new definition of L-series without neglecting the singular and branch points, which is a natural generalization of Lang's one and Weil's one given in the case of curves in [**9**]. Ishida also treated L-series in a different way in [**6**]. It will be seen that our definition and the one given in [**6**] are the same one.

On the other hand Sampson and Washnitzer [**8**] obtained a functional equation of the zeta-function of the non-singular variety U under some assumption. Using the same assumption as that used in [**8**], we shall deduce a functional equation of our L-series for the Galois covering V/U when U is a non-singular variety. When U is a curve, it is obtained by Weil in [**9**]. When U is an abelian variety with the abelian Galois group $\mathfrak{G}$, the same result is obtained by Ishida in [**5**]. Thus our L-series will seem to be a satisfactory one.

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§ 1. Galois coverings defined over a finite field k.

Let $\pi: U\to V$ be a Galois covering of degree n, defined over a finite field k with q elements.[1] In the following we shall assume that U and V are normal, projective varieties of dimension r. Let $\alpha, \sigma, \tau, \cdots$, be the automor-

1) For the definition of a Galois covering of an algebraic variety, see Lang [8].