Journal of the Mathematical Society of Japan Vol. 12, No. 2, April, 1960

On Witt ring of quadratic forms.

By Yoshio NAKAMURA

(Received Dec. 23, 1958)
(Revised June 22, 1959)

§1. **Introduction.** Witt has proved that the classes of 'ähnlich' forms over a field k, which has characteristic not 2, form a ring (Witt [**2**]). This ring will be called *Witt ring* over k, in this paper. We shall consider the structure of Witt ring. Our results will be shown in theorem 1 for a finite field, in theorem 2 for a complete field with respect to a discrete non-Archimedean valuation, whose residue class field is finite and of characteristic not equal to 2, where Witt ring over that field is related to Witt ring over the residue class field, and in theorem 3 for an algebraic number field of finite degree over the rational number field.

I am quite indebted to Mr. A. Hattori, who has given kind help throughout.

§2. **Preliminaries.** In the first place, Eichler's formulation of *Witt group* in terms of metric spaces will be shown as follows (Eichler [**1**]):

Let k be a fixed commutative field of characteristic not 2, then a vector space R over k is made into a metric space by defining the *(inner) product* $\xi\eta$ of two vectors ξ, η, such that $\xi\eta$ is in k and

1. $\xi\eta=\eta\xi$,
2. $(\xi+\eta)\zeta=\xi\zeta+\eta\zeta$,
3. $(x\xi)\eta=x(\xi\eta)$, $x\in k$.

We consider only finite dimensional metric spaces over k. If $\{\iota_1, \cdots, \iota_n\}$ is a basis of R over k (in this case we write $R=k(\iota_1, \cdots, \iota_n)$), the square ξ^2 of $\xi=\sum_{i=1}^{n} x_i\iota_i$, $x_i\in k$, is a quadratic form

$$f=\sum_{i,j=1}^{n} f_{ij}x_ix_j, \qquad (f_{ij}=f_{ji}\in k)$$

in x_i, where $f_{ij}=\iota_i\iota_j$. f is called a *fundamental form* of R, and we denote this by $f\cdots R$. Conversely, every quadratic form f over k is a fundamental form of some space R.

Spaces R are always assumed to be *semi-simple*, namely for every vector $\xi\neq 0$ in R there is a vector η such that $\xi\eta\neq 0$, in other words, if $f\cdots R$, the determinant $|f_{ij}|$ of the matrix (f_{ij}) of coefficients of f is not zero.