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Domains spread on a complex space.

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Let X and Y be two connected complex analytic manifolds, and let φ be a holomorphic mapping of X into Y. If φ is a local homeomorphism, we usually call the triple (X, φ, Y), or simply the pair (X, φ), a domain spread on Y by φ. The general theory of spread domains has been established by H. Cartan[1] and others. In the classical theory of functions in several complex variables, only these spread domains have been considered. A general tendency at the present time is, however, to investigate the so-called "complex spaces" and the behavior of functions on them. It is natural to include the algebroidal functional elements in the domain of holomorphic prolongation of holomorphic functions. Thus we are led to introduce the notion of "ramified spread domains".

We shall generalize the notion of spread domains as follows: Let X and Y be two connected normal complex spaces of the same dimension in the sense of H. Cartan[2], and let φ be a holomorphic mapping of X into Y; if φ is non-degenerate at every point of X, that is, if the fiber $\varphi^{-1}(\varphi(x))$ of φ through x is a discrete set in X for every point x of X, then we call (X, φ, Y) or (X, φ) a domain spread on Y by φ. In § 1 we recall briefly the notion of complex spaces and make some remarks for later use. In § 2 the space of (holomorphic-) algebroidal jets of a complex space into another is introduced as a generalization of the space of holomorphic jets. § 3 is devoted to the general theory of spread domains; as in the classical case of "unramified spread domains", the maximal holomorphic prolongation and the intersection of spread domains are defined. Using a space of algebroidal jets we prove the existence theorem of the maximal holomorphic prolongation of a given spread domain with respect to a family of holomorphic mappings. The exposition in § 3 is made after the manner of H. Cartan's seminar.

1) cf. [1].
2) cf. [2], VIII bis.