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On the change of rings in the homological algebra.

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The object of this note consists in making a response partly to Massey's problem 22 ([4]), generalizing the problem from the case of the homology of groups to the case of the homological algebra. Thus we shall study the change of rings $\varphi: \Lambda \rightarrow \Gamma$ in the homological algebra by the use of algebraic mapping cylinder as was indicated in Massey's paper. In our description, we shall make free use of the notations of the book of H. Cartan and S. Eilenberg ([1]).

In § 1, we define the algebraic mapping cylinder by analogy of the topological mapping cylinder (cf. p. 73, [1]). In § 2, we introduce new functors Tor^{φ} and Ext_{φ}, which have similar properties as the absolute Tor- and Ext-functors, as will be shown in § 3.

The functors Tor^{φ} and Ext_{φ} yields a "relative" theory for the change of rings $\varphi: \Lambda \rightarrow \Gamma$. Another "relative" theory of the homological algebra was discussed in [3]. According to what is announced in the same paper, our problems seem to be also investigated by M. Auslander. But I have not yet access to his results. In § 4, we shall consider in particular the relative cohomology group of dimension 2, $H^2(\mathfrak{G}, \mathfrak{K}: M)$ (M being a $\mathfrak{G}$-module) of a group $\mathfrak{G}$ and its subgroup $\mathfrak{K}$, and bring it in relation with the classes of group extensions of $\mathfrak{G}$, which are trivial over $\mathfrak{K}$.

The author has in view to analyse further the relations between the relative homology of a pair $(\mathfrak{G}, \mathfrak{K})$ and the homology of factor group $\mathfrak{G}/\mathfrak{K}$ in case $\mathfrak{K}$ is normal. We can readily see that if $H^r(\mathfrak{K}: M) = 0$, for $0<r<n$, then $H^n(\mathfrak{G}, \mathfrak{K}: M) \approx H^n(\mathfrak{G}/\mathfrak{K}: M^{\mathfrak{K}})$ by our reduction theorem 3.4* in § 3 and by the exact sequence of Hochschild and Serre ([2]). These topics will be treated in a forthcoming paper.

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