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Locally convex lattices.

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Since normed vector lattice was considered first by L. Kantorovitch [1] (the numbers in brackets refer to the list of References at the end of this paper), the theory of Banach lattices has undergone a considerable development, but very little effort seems to have been made to extend this theory to more general topological lattices (see Nakano [3]. On a similar situation in the theory of topological algebra, see Michael [1]). On the other hand, the theory of Banach spaces was considerably extended in its range of application by introducing the notion of locally convex spaces (Mackey [1], [2]; Dieudonné et Schwartz [1]; Bourbaki [1], [2], [3]).

The purpose of this paper is to generalize the theory of normed vector lattices in an analogous fashion, by introducing the concept of locally convex lattices.

A locally convex lattice (for Definition, see § 1) is a locally convex Hausdorff space over the reals as well as a vector lattice such that whenever a net (a directed system) $\{x_\lambda\}$ converges to 0 and $|x_\lambda| \geqq |y_\lambda|$ for each λ, then $\{y_\lambda\}$ converges to 0. If a locally convex lattice is an $\mathfrak{L}\mathfrak{F}$-space, it will be called an $\mathfrak{L}\mathfrak{F}$-lattice. An $\mathfrak{F}$-lattice is a metrizable and topologically complete locally convex lattice.

This paper is divided into six sections, the first of which is concerned with the definition and fundamental properties of locally convex lattices. Every locally convex lattice E is generated, in a certain sense, by normed vector lattices $\{E_\alpha\}$ (Theorem 1.3, 1.4) and then most of problems about locally convex lattices can be reduced to similar ones about normed vector lattices. Thus, in § 3, we shall study the relations between E and $\{E_\alpha\}$.

§ 2 is concerned with the conjugate spaces and duals of locally convex lattices.

The completion $\hat{E}$ (as a locally convex space) of any locally