

On Pontrjagin classes and homotopy types of manifolds.

By Itiro TAMURA

(Received March 15, 1957)

1. Introduction.

In the present paper we shall obtain manifolds of the same homotopy type with different Pontrjagin classes, belonging therefore to different classes in the sense of diffeomorphism (i. e. differentiable homeomorphism).

This reveals the Pontrjagin classes as no homotopy invariants and the problem of "topological invariance of Pontrjagin classes," except for mod 2 and mod 3, as not provable by means of homotopy invariants, such as (co)homology groups, homotopy groups, Steenrod operators etc.

In section 2 of this paper we define some sphere bundles over spheres and determine the homotopy types of them by method of A. Dold. In section 4, Pontrjagin classes of these bundles are calculated from the Chern classes of associated bundles by the obstruction theory prepared in section 3. Section 5 is devoted to the description of the cohomology groups of total spaces of bundles. The principal tool here is the Gysin exact sequence.

In section 6, C^∞ -manifolds are defined from the bundles and their Pontrjagin classes are computed. Our final results are exposed in section 7.

We use in this paper the results on the homotopy groups of spheres and classical groups which can be found, for example, in Steenrod [11], Borel et Serre [1], Serre [10].

The author wishes to express his hearty thanks to his friends T. Nakamura, A. Hattori for their kind discussions and valuable suggestions, and also to Professor S. Iyanaga for his constant encouragement.