Journal of the Mathematical Society of Japan Vol. 9, No. 2, April, 1957

Groups of projective transformations and groups of conformal transformations.

By Shigeru ISHIHARA

(Received Feb. 2, 1957)

In 1928, M. S. Knebelman [7] proved that a group of projective transformations in an n-dimensional affinely connected manifold preserves a projectively related affine connection if the group is of order $r \leqq n$. In this respect, it seems to be interesting to ask whether a group of conformal transformations of a Riemannian metric preserves or not another Riemannian metric. In § 1 we shall show that a group G of projective transformations of an affine connection leaves another projectively related affine connection invariant if G is compact. For a transitive group G we shall further prove that the same remains valid, if the isotropy group of G is compact, or, if the identity component of the linear isotropy group of G is irreducible and the space is projectively non-flat. In § 2 we shall obtain, concerning groups of conformal transformations, some results analogous to those proved in § 1.

On the other hand, the compactness, the completeness or the irreducibility of a Riemannian manifold implies strong restrictions on affine, conformal or isometric transformations [1, 3, 6, 8, 10, 11, 19, 20]. In this respect, in § 3 we shall study groups of projective transformations preserving the Ricci tensor in an affinely connected manifold and obtain the fact that such groups are affine in a space, which is complete, or, whose homogeneous holonomy group has no invariant hyperplane. In § 4 such groups will be discussed in a complete or compact Riemannian manifold. In § 5 we shall study groups of conformal transformations leaving the Ricci tensor invariant in a complete or compact Riemannian manifold and obtain some results analogous to those proved in § 4.

The last section is devoted to the proof of a lemma used in § 1 concerning groups of affine motions of the ordinary affine space.