Journal of the Mathematical Society of Japan Vol. 9, No. 1, January, 1957

On decomposable symmetric affine spaces.

By Atsuo FUJIMOTO

(Received Oct. 5, 1956)

§ 1. Decomposable spaces

Consider two affinely connected spaces without torsion A_p and A_{n-p} of the dimension p and $n-p$ respectively. Denote by $\Gamma^{i^1}_{j^1k^1}(x^{l^1})$ and $\Gamma^{i^2}_{j^2k^2}(x^{l^2})$ the connections, (x^{i^1}) and (x^{i^2}) the coordinates on A_p and A_{n-p} respectively. As to the ranges of indices we shall adopt the following convention $i, j, k, l=1,\cdots,n$; i^1, j^1, k^1, l^1 (indices of the first kind) $=1,\cdots,p$; i^2, j^2, k^2, l^2 (indices of the second kind) $=p+1,\cdots,n$.

The n-dimensional affinely connected space A_n with coordinates (x^{i^1}, x^{i^2}) and the connection $\tilde{\Gamma}^i_{jk}$ will be called the *product space* of A_p and A_{n-p}, if the components of the connection with the indices of different kind vanish and $\tilde{\Gamma}^{i^1}_{j^1k^1}=\Gamma^{i^1}_{j^1k^1}(x^{l^1})$, $\tilde{\Gamma}^{i^2}_{j^2k^2}=\Gamma^{i^2}_{j^2k^2}(x^{l^2})$. In this case A_n is said to be *decomposable*, and the coordinates (x^{i^1}, x^{i^2}) are called a code. When (y^{i^1}) and (y^{i^2}) are normal coordinates on A_p and A_{n-p} respectively, then (y^{i^1}, y^{i^2}) is a normal code on A_n ([1]).

An object defined on a decomposable A_n is said to be *breakable* if its components with the indices of different kind are all zero with respect to a code. If an object is breakable and its components with indices of the same kind depend, in any code, only on the variables of that kind, then the object is called a *product object*.

§ 2. Symmetric affine space

An n-dimensional affinely connected space A_n without torsion is said to be *symmetric* in Cartan's sense if the reflexion about any point in A_n is an affine collineation. An A_n with connexion Γ^i_{jk} is symmetric if and only if the first covariant derivative of the curvature tensor vanishes, i. e.

$$R^i_{jkl;m}=0\,,$$

where

$$R^i_{jkl}=\Gamma^i_{jk,l}-\Gamma^i_{jl,k}+\Gamma^h_{jk}\Gamma^i_{hl}-\Gamma^h_{jl}\Gamma^i_{hk}\,;$$