Journal of the Mathematical Society of Japan Vol. 9, No. 1, January. 1957

Intrinsic character of minimal hypersurfaces in flat spaces.

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(Received Aug. 3, 1956)

Introduction.

A minimal variety in a Riemannian space is defined as a variety which realizes an extremal of the volume integral, and is characterized, from a stand-point of the differential geometry in the small, by the property that the mean curvature vanishes. Although many properties of such spaces immersed in an enveloping space are known, it seems to me that their intrinsic properties have not been yet discussed.

In this paper we investigate the *intrinsic properties of minimal hypersurfaces in flat spaces.* At the beginning of Section 3, the tensors $S_{p)ij}$ are defined in terms of the curvature tensor and they play an important rôle throughout the paper. The first two sections are devoted to explain how the tensors are derived. In Section 3, by means of these tensors, the coefficients of the second fundamental form are written in terms of the curvature tensor, and then, from the Gauss equation, we obtain the identities which are satisfied by the components of the curvature tensor of a minimal hypersurface.

In Section 4, the classification theorem of minimal hypersurfaces is obtained with the aid of the tensors $S_{p)ij}$, and then we get the imbedding theorem of a Riemannian space as a minimal hypersurface in a flat space.

There exists a special class of minimal hypersurfaces, which will be called to be of type M^{∞} and for which we can not determined the coefficients of the second fundamental form by the general method used in Section 3. Any minimal surface of ordinary space belongs to this class. In the final two sections, we shall treat the Einstein spaces, conformally flat spaces, and 3-dimensional spaces as simple examples of such an exceptional case.