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On conformally curved Riemann spaces V_n, $n \geqq 6$, admitting a group of motions G_r of order $r > n(n+1)/2-(3n-11)$.

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Synopsis

The conformal curvature tensor $C_{\lambda\mu\nu\omega}$ of a Riemann space V_n, $n \geqq 6$, admitting a group of motions of order $r > n(n+1)/2-(3n-11)$ is studied with the use of tensor calculus. The form of $C_{\lambda\mu\nu\omega}$ is obtained by virtue of the fact that the equations $XC_{\lambda\mu\nu\omega}=0$ can contain at most a certain number of linearly independent equations. The $C_{\lambda\mu\nu\omega}$ is in general of the form

$$
\begin{aligned}
C_{\lambda\mu\nu\omega} &= C[\delta_{\lambda\omega}\delta_{\mu\nu}-\delta_{\lambda\nu}\delta_{\mu\omega}] \\
&-((n-1)/2)C[\delta_{\lambda\omega}(A_\mu A_\nu+B_\mu B_\nu)+\delta_{\mu\nu}(A_\lambda A_\omega \\
&+B_\lambda B_\omega)-\delta_{\lambda\nu}(A_\mu A_\omega+B_\mu B_\omega)-\delta_{\mu\omega}(A_\lambda A_\nu+B_\lambda B_\nu)] \\
&+((n-1)(n-2)/2)C[A_\lambda A_\omega B_\mu B_\nu+A_\mu A_\nu B_\lambda B_\omega \\
&-A_\lambda A_\nu B_\mu B_\omega-A_\mu A_\omega B_\lambda B_\nu]
\end{aligned}
$$

with $A_\alpha A_\alpha=B_\alpha B_\alpha=1$, $A_\alpha B_\alpha=0$. But for $n=6, 8$ some other form is also possible.

§ 1. Introduction.

It is well known [1, 3, 9][1] that we have the

THEOREM 1. *If an n-dimensional Riemannian space admits a group of motions of order $n(n+1)/2$, then, the space is of constant curvature.*

As for the Riemannian spaces which are not of constant curvature, we have the following theorems.

THEOREM 2. *An n-dimensional Riemannian space for $n>2$, $n \neq 4$,*

1) Numbers in brackets refer to the references at the end of the paper.