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Remarks on Boolean functions II.[1]

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1. Introduction.

This paper continues our remarks on Boolean functions [7][2]. In the present paper we are concerned with the groupoids [5] arising from functions of two variables and with the factorization of general functions. Some of the matters in Sections 3 and 4 have been partially discussed previously in [3] and [9], respectively. The Boolean algebra, B, considered throughout is strictly arbitrary.

2. Preliminaries.

Let B be a Boolean algebra [1] with meet, join, and complement indicated by $x\wedge y, x\vee y$, and x^*, respectively. We shall also employ the ring notation [10], $x+y$ and xy, where these denote sum and product, respectively. One recalls [10]:

$$x+y=(x\wedge y^*)\vee(x^*\wedge y)$$
$$xy=x\wedge y$$
$$x\vee y=x+y+xy\,.$$

The first and last elements of B (additive and multiplicative identities in the ring) will be denoted by 0 and 1, respectively.

One recalls [1] that any Boolean function, $f(x,y)$, of two variables over B may be written in its disjunctive normal form:

$$(\dagger)\ f(x,y)=(a\wedge x\wedge y)\vee(b\wedge x\wedge y^*)\vee(c\wedge x^*\wedge y)\vee(d\wedge x^*\wedge y^*)\,.$$

The standard ring form of $f(x,y)$ is

1) Presented to the Mathematical Association of America, Athens, Georgia, March 1956.
2) Numbers in square brackets refer to the list of references concluding the paper.