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Affine transformations in an almost complex manifold with a natural affine connection

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In an almost complex manifold there exists an affine connection in which the almost complex structure is covariant constant [2][1]. In this paper such an affine connection, not necessarily symmetric, is said to be *natural*. When we speak of an almost complex manifold, we shall always bear a fixed natural affine connection in mind. An affine transformation in an affinely connected manifold is, roughly speaking, a differentiable transformation leaving the affine connection invariant [6, 12].

It might be of interest to ask whether an affine transformation preserves the almost complex structure or not, and if not, then what the structure of the manifold is. In this respect, A. Lichnerowicz [5] has recently proved that in an irreducible Kählerian manifold of dimension $2n$ the largest connected group of isometries preserves the almost complex structure[2] if n is odd or if n is even and the Ricci curvature tensor does not vanish. J. A. Schouten and K. Yano [10] have also proved the same result for the pseudo-Kählerian manifold.

We shall prove that in an irreducible almost complex manifold if the largest connected group of affine transformations does not preserve the almost complex structure, then n is even and the homogeneous holonomy group is contained in the real representation of the quaternionian linear group. Furthermore in this case a homomorphism of the group of all affine transformations into the special orthogonal group of three dimensions will be obtained. Our result generalizes the results of A. Lichnerowicz and J. A. Schouten and K. Yano.

In a complex manifold there exists, as is well-known, a symmetric

1) See the Bibliography at the end of the paper.
2) If the manifold is compact the theorem holds true without any other restriction [5].