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A note on close-to-convex functions.

By Yoshikazu MIKI

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I. Introduction

G. Szegö [1] proved the following theorem on the partial sums of the normalized schlicht functions in the unit circle;

Let the function

$$f(z)=z+\sum_{\nu=2}^{\infty}a_{\nu}z^{\nu}$$

be analytic and schlicht in $|z|<1$. *Then any one of the partial sums*

$$z+\sum_{\nu=2}^{n}a_{\nu}z^{\nu} \qquad (n=2, 3,\cdots)$$

is also schlicht in $|z|<\frac{1}{4}$, *and the constant* $\frac{1}{4}$ *can not be replaced by any greater one.*

Szegö [1] proved also that in this theorem the word 'schlicht' may be replaced by 'star-shaped with respect to the origin' or by 'convex', both in the hypothesis and in the conclusion in corresponding manner.

In this note, we shall prove a similar theorem for the class of close-to-convex functions defined by W. Kaplan [2].

We call an analytic function $f(z)$ close-to-convex for $|z|<R$, if there exists a function $\varphi(z)$, convex and schlicht for $|z|<R$, such that $f'(z)/\varphi'(z)$ has positive real part for $|z|<R$. This function $\varphi(z)$ will be called an *associate* function to the close-to-convex function $f(z)$, and we shall call $f(z)$ close-to-convex *with respect* to $\varphi(z)$, when it is needed to indicate an associate function. Close-to-convex functions in the unit circle will be simply called close-to-convex functions.

Thus close-to-convex functions are clearly schlicht for $|z|<1$, and the class of these functions includes the classes of star-shaped functions and convex functions for $|z|<1$, ([2]). We aim at proving