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Analytic vector functions of several complex variables.

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In this paper, we shall consider a system of k functions, which we shall call a vector function following Bochner-Martin*), of k complex variables. We shall show that various theorems of the theory of functions of a complex variable can be generalized to the case of vector functions. In our previous paper [2] in collaboration with Prof. S. Ozaki, we have established the expansion theorem and the estimation of derivatives for vector functions in polycylindrical domains. Now we shall study such functions in more general domains.

In § 1, we shall prove the expansion theorem and the residue theorem, and give a representation of derivatives and coefficients.

In § 2, we shall consider bounded vector functions, and generalize Gutzmer's inequality, Schur's estimation of coefficients and Landau-Dieudonné's theorem concerning the univalence radius of a hypersphere, etc. The estimation of coefficients was given by E. Peschl and F. Erwe [3] in the case of systems of functions of a complex variable. About the univalence radius some results were obtained by S. Takahashi [4].

In § 3, we shall generalize the argument principle in the case of a complex variable and obtain a formula giving the number of zero points of vector functions. The set of zero points of a single function of several complex variables forms a manifold, but the zero points of vector functions are in general isolated, so that we can speak of the number of them.

§ 1. General considerations

1. *Distance and norm.* We introduce the real coordinates $x_1, y_1, \cdots, x_k, y_k$ in the $2k$-dimensional Euclidean space and put $z_j = x_j + iy_j$,

*) See Bochner-Martin [1], Chap. VIII. § 5.