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The linear equivalence theory of cycles and cycles of dimension zero on abelian varieties.

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The theory of linear equivalence relation in the algebraic geometry has been hitherto developed principally for divisors on varieties. In the present paper an attempt is made to generalize this theory to the case of cycles of arbitrary dimensions. In § 1, we shall define such equivalence and show that all properties announced by A. Weil in his book "Foundations of Algebraic Geometry" as necessary properties of such equivalence are satisfied.

Linear equivalence of two cycles X, Y will be denoted by $X \sim Y$. Functions $f(X)$ (generally with rational integral values) of cycles with the property: '$X \sim Y$ implies $f(X)=f(Y)$' will be called *linear invariant*. For instance, the ranks of complete linear systems or the indices of speciality of divisors on a curves are linear invariant.

In § 2, we shall deal with cycles $\mathfrak{a}$ of dimension zero on a product variety of complete non-singular curves Γ_i, $1 \leqq i \leqq n$, and introduce a linear invariant $l(\mathfrak{a})$ in generalization of the rank of complete linear system in case $n=1$.

In § 3, we consider cycles $\mathfrak{a}$ of dimension zero on an abelian variety. Using the result of § 2, we define a linear invariant $l(\mathfrak{a})$ and $\beta_i(\mathfrak{a})$, $1 \leqq i \leqq n$. We define further the index d and the pseudo-genus g of an abelian variety. $l(\mathfrak{a})$ can be written in the form:

$$l(\mathfrak{a})=d^n(\deg \mathfrak{a})^n+d^{n-1}\beta_1(\mathfrak{a})(\deg \mathfrak{a})^{n-1}+\cdots+d^{n-i}\beta_i(\mathfrak{a})(\deg \mathfrak{a})^{n-i}+\cdots$$

$+\cdots+\beta_n(\mathfrak{a})$, and if $\deg \mathfrak{a}$ is sufficiently large, $\beta_i(\mathfrak{a})$ becomes a constant $(-1)^i \binom{n}{i} (g-1)^i$.

In § 4, we prove the birational invariance of these $l(\mathfrak{a}), \beta_i(\mathfrak{a}), g$, and d.

As to the notations and terminologies, we follow the usage in Weil [1], [2], [3].