Journal of the Mathematical Society of Japan Vol. 8, No. 2, April, 1956

On birational invariance of classical groups.

By Takashi ONO

(Received Feb. 25, 1956)

In the present paper, we define the notion of birational isomorphisms for algebraic groups by making use of the rational representations treated in Chevalley's book [1],[1] and formulate the following problem. Given a family $\mathfrak{F}$ of algebraic groups, is it birationally invariant (that is, does $\mathfrak{F}$ contain every algebraic group which is birationally isomorphic with a group in $\mathfrak{F}$)?

Here we take as $\mathfrak{F}$ the family composed of classical groups,[2] and shall show that the answer is affirmative if the field of definition of the groups is of characteristic zero and the dimension of the vector space on which groups operate is large enough. The proof depends on Weyl's representation theory and Dieudonné's structure theory of classical groups. We don't know whether the same assertion is true or not for algebraic groups defined over a field of characteristic p.

The writer wishes to express his thanks to Mr. N. Iwahori and Mr. J. Hano for their discussions with him.

§ 1. Birational isomorphism.

Let V be a finite dimensional vector space over an infinite field K,[3] and let E be the vector space of endomorphisms of V. We shall say that two algebraic groups G and G' on V are birationally isomorphic if there is an isomorphism ρ of G onto G' such that ρ and

---

1) [1] p. 101, Definition 4. In this paper, we shall freely use definitions and results in [1].

2) By classical groups we mean the following groups operating on a vector space V: $GL(V)$, $SL(V)$, $Sp(V,f)$, f being a non-degenerate skew symmetric bilinear form on V, $O(V,f)$, f being a non-degenerate symmetric bilinear form on V, and $SO(V,f)=SL(V)\cap O(V,f)$. we denote their Lie algebras by the corresponding small German letters.

3) Hereafter, by a field we shall always mean a field with infinitely many elements.