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On an absolute constant in the theory of quasi-conformal mappings.[1]

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1. A topological mapping $w=T(z)$ of a planer region D onto another such region Δ is called a quasi-conformal mapping with the parameter of quasi-conformality K, or, simply, a K-QC mapping, if

(i) it preserves the orientation of the plane; and

(ii) for any quadrilateral $\Omega(z_1, z_2, z_3, z_4)$ contained in D together with its boundary, the inequality

$$\operatorname{mod} T(\Omega(z_1, z_2, z_3, z_4)) \leqq K \operatorname{mod} \Omega(z_1, z_2, z_3, z_4),$$

holds, where K is a constant $\geqq 1$. (See Mori [3], [4] and also Ahlfors [1].)

Let $w=T(z)$ be a K-QC mapping of $|z|<1$ onto $|w|<1$ such that $T(0)=0$. Then, as is already known, this mapping can be regarded as a topological mapping of $|z|\leqq 1$ onto $|w|\leqq 1$. (See Ahlfors [1], Mori [3], [4].) And, if z_1, z_2 are arbitrary two points on $|z|\leqq 1$, we have

$$|T(z_1)-T(z_2)| \leqq C|z_1-z_2|^{\frac{1}{K}}, \tag{1}$$

where C is a numerical constant.

To the author's knowledge this was first proved by Yûjôbô [6], though under a narrower definition than that given above, the author proved it with $C=48$. (Mori [3], [4]). Though with C depending on K, ($C=12^{K^2}$), Ahlfors proved (1) under the same definition. Further, Lavrentieff is reported to have proved (1) in a paper to which the author has not access (Lavrentieff [2]), so the author does not know with what C and under what definition Lavrentieff proved it.

1) The author of this paper, A. Mori suddenly died on July 5, 1955 at the age of 30. This paper was edited by Z. Yûjôbô after a manuscript of A. Mori (written in Japanese) found after his death.