Journal of the Mathematical Society of Japan Vol. 8, No. 1, January, 1956

# On the fundamental conjecture of *GLC* III.

By Gaisi TAKEUTI

(Received Nov. 11, 1954)

This paper is a continuation of [1] and [2]. We use the same notions and the notations as in these papers. See in particular [1] as to the meaning of the fundamental conjecture. We have proved this conjecture under several conditions in [1], [2]. In this paper, we shall prove it under some other conditions.

## § 1. Formulation of the theorem.

Until at the end of Appendex, the logical symbols $\exists$ and $\vee$ are not used. In this section we introduce some new notions and notations.

1.1. A formula in a proof-figure and a logical symbol in a formula

We shall speak of a 'formula in a proof-figure', when the formula is considered together with the place where it occupies in the proof-figure. Let $A$ and $B$ be two formulas in a proof-figure $\mathfrak{P}$. Then $A$ is equal to $B$ if and only if $A$ is in the same place as $B$ in $\mathfrak{P}$. We shall also speak of logical symbol in a formular or in a proof-figure sequence and inferences etc. in a proof-figure in analogous meanings. We use the symbols $\sharp$, $\natural$ etc. as metamathematical variables to represent logical symbols in a formula or in a proof-figure.

1.2. Semi-formula, quasi-formula.

A figure of the form $H(x,\cdots,y,\varphi,\cdots,\psi)$ with bound variables $x,\cdots,y$ and bound $f$-variables $\varphi,\cdots,\psi$ is called a semi-formula, if rnd only if $H(a,\cdots,b,\alpha,\cdots,\beta)$ obtained from $H(x,\cdots y,\varphi,\cdots,\psi)$ by substituting free variables $a,\cdots,b$ and free $f$-variables $\alpha,\cdots\beta$ for $x,\cdots,y$ and $\varphi,\cdots,\psi$ is a formula and $x,\cdots,y$, $\varphi,\cdots,\psi$ are difierent from each other and are not contained in $H(a,\cdots,b,\alpha,\cdots,\beta)$.

If $\{x,\cdots,y\}H(x,\cdots,y)$ is a formula with argument-places, then $H(x,\cdots,y)$ is clearly a semi-formula.