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A generalization of the principal ideal theorem

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The purpose of this paper is to give a cohomology-theoretical description of a generalized principal ideal theorem. The definitions and the notations in this paper are borrowed from C. Chevalley's lecture notes at Nagoya University [1].

1. Let G be a finite group, and S be an automorphism of the group G. The image of an element $\sigma\in G$ by S will be denoted by $S(\sigma)$. Let H be the invariant subgroup of G, which is generated by all the elements $S(\sigma)\sigma^{-1}$, $\sigma\tau\sigma^{-1}\tau^{-1}(\sigma, \tau G)$. Then H is an S-invariant subgroup of G, and G/H is abelian.

Let A be a G-module. We shall denote a submodule of A which is generated by all the elements $(1-\sigma)a$ $(\sigma\in G, a\in A)$ by I_GA. Especially, if A is the group ring $Z(G)$ over the integral domain Z of all rational integers, the submodule $I_GZ(G)$ will be denoted by I_G. We shall use analogous symbols concerning subgroups of G.

2. We shall consider, in this section, certain mappings of the cohomology groups of G.

Let $x, y\in I_G$, and $n\in Z$. Then, $x\otimes y\otimes n\rightarrow x\otimes ny$ defines an isomorphism $\psi_G: H^{-2}(G, Z)\rightarrow H^{-1}(G, I_G)$. We have also an isomorphism $\psi_H: H^{-2}(H, Z)\rightarrow H^{-1}(H, I_H)$.

Let A be a G-module, and $\tau\in G, a(\tau)\in A$ such that $\sum a(\tau)=0$. Then, $\sum\tau\otimes a(\tau)\rightarrow a(e)$, where e is the unit element of G, induces an isomorphism $H^{-1}(G, A)\rightarrow A^{G\rightarrow 0}/I_GA$. Especially, if $A=I_G$, we have an isomorphism $\varphi_G: H^{-1}(G, I_G)\rightarrow I_G/I_GI_G$, and also, $\varphi_H: H^{-1}(H, I_G)\rightarrow I_G^{H\rightarrow 0}/I_HI_G$.

We have also an isomorphism $\Phi: H^{-1}(H, I_H)\rightarrow H^{-1}(H, I_G)$ (cf. [1] Theorem 7.1).

Let j_{-r} be the injection mapping $H^{-r}(H, I_G)\rightarrow H^{-r}(G, I_G)$, $r=1,2$. Then, $\varphi_G j_{-1}\varphi_H^{-1}$ maps $I_G^{H\rightarrow 0}/I_HI_G$ into I_G/I_GI_G, and the kernel is the subgroup $(I_{G'}, I_HI_G)/I_HI_G$ of the group $I_G^{H\rightarrow 0}/I_HI_G$, where G' is the commutatorsubgroup of G.

The ideal I_G of $Z(G)$ is generated by all the elements $1-\sigma$, $\sigma\in G$, and each element of I_G is described as $\sum a(\sigma)(1-\sigma)$, where $a(\sigma)\in Z$.