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The predicate calculus with ε-symbol.

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The purpose of this paper is to prove the following theorem:

"If a formal axiom system represented by formulae in the ordinary predicate calculus is consistent in the ordinary predicate calculus, it is consistent also in the predicate calculus with ε-symbol."[1]

By the ordinary predicate calculus we mean here Gentzen's 'Kalkül LK[2]', and what we call here 'ε-symbol' means the logical symbol 'ε' used in representing the quantifier 'εx' which was originally proposed by Hilbert and named 'transfinite logische Auswahlfunktion'. When $F(x)$ represents a proposition containing the variable x for an individual, as long as there exists at least such an x as makes $F(x)$ true, $\varepsilon x F(x)$ indicates such an x as makes $F(x)$ true. And if there exists no x such as makes $F(x)$ true, $\varepsilon x F(x)$ means an arbitrary individual.[3]

For obtaining the predicate calculus with ε-symbol (of first order), it is sufficient, as is well-known, to adjoin the logical axiom schema

$$F(a) \rightarrow F(\varepsilon x F(x))$$

and appropriate rules of inference to the propositional calculus. But, for the sake of convenience, we now use as the predicate calculus with ε-symbol the logical system obtained from the ordinary predicate calculus[4] by adjoining the above logical axiom schema to it.

In an Appendix, we shall consider the ε-symbol on propositions.

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§ 1. Terminologies and symbols.

1.1. 'Term' and 'formula'.

1.11. DEFINITION:

1.111. A free variable is a *term.*

1.112. If $\mathfrak{t}_1, \dots, \mathfrak{t}_n$ are *terms*, and $\mathfrak{f}(*, \dots, *)$ is a function of n argu-