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On geometry of numbers.

By L. A. SANTALÓ

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1. Introduction. In a recent paper Tsuji [3] has given some theorems which may be considered as a generalization to Fuchsian groups of the classical theorems of Blichfeldt [1] and Minkowski in the geometry of numbers. In both cases, however, the generalization is restricted to circular domains.

The object of this paper is to carry this generalization further. First, by considering more general groups than the Fuchsian groups; second, by considering a kind of domains more general than the circular domains.

2. A preliminary integral formula. Let S be a space of points in which operates a transitive group of transformations G. For a point $P \in S$ and an element $s \in G$ we denote with sP the transform of P by s. The elements s of G with $sP_0=P_0$ for a given point P_0 of S form a subgroup G_0 of G. Let G_1 be a simply-transitive subgroup of G. Then we have obviously $G_1G_0=G$, $G_1 \frown G_0=\{e\}$ (unit group), and we can identify G_1 with the homogeneous space G/G_0 or with the space S by assigning $x \in G_1$ to $xG_0 \in G/G_0$ or to $xP_0 \in S$. G may be then considered as operating on G_1, as well as on S, in the following manner. Let s, $t \in G$, $x \in G_1$, then sxt, properly an element of G, is identified with the element of G_1 corresponding to $sxtG_0$ in G/G_0. We assume now G_1 to be locally compact and provide G/G_0 resp. S with the same topology as that of G_1. We assume further that G_1 is unimodular, i. e. that the left invariant measure of G_1 is also right invariant; and that this measure is also invariant with respect to G, so that we have $m(H)=m(sHt)$ for a point-set H in G_1 and any $s, t \in G$. This may be symbolically written as follows ([4, 34]):

(2.1) $dx=dx^{-1}$, $dx=d(sxt)$ for any $s, t \in G$,

(2.2) $m(H)=\int_{G_1}\varphi(x)dx$,

where dx denotes the element of volume of G_1 and $\varphi(x)$ the charac-