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On fibre spaces in the algebraic number theory.

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Recently A. Weil ([4], [5]) has introduced successfully the concept of the fibre space into the algebraic geometry, and proved the important classification theorem. In this paper we try to establish an analogous theory on the algebraic number field.

Starting from any algebraic number field k, we shall begin with constructing an analogue $S(k)$ of the algebraic curve in § 1, and then define in § 2 the "W-variety," corresponding to the algebraic variety in the algebraic geometry. After having defined the direct product of W-varieties, and "group W-variety" in § 3, we shall introduce the fibre spaces over the W-varieties, and prove the existence theorem and the classification theorem in the last two paragraphs. A certain group of fibre spaces corresponding to that of line bundles ([3], [5]) turns out to be isomorphic with the classical group of "Strahlklasse" in k, if k has a finite degree.

We shall here concern ourselves exclusively with the multiplicative structure of algebraic number fields. In order to take the additive structure of these fields also in account, it seems necessary to have recourse to the concept of "faisceaux" or some other new ideas.

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§ 1. Construction of an analogue of the algebraic curve.

Let k be any algebraic number field of a finite or an infinite degree fixed once for all. We denote by $k_\lambda (\lambda \in \Lambda)$ the fields which are subfields of k and have finite degrees over the rational number field, and we define a semi-order $\lambda < \mu$ for $k_\lambda \subset k_\mu$ in Λ, then Λ becomes a directed set.