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On the kernel of semigroups.

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The structure of the kernel of finite semigroups was studied by Suschkewitsch [1], and his study has been extended to bicompact semigroups by Numakura [2]. In the latter case, the set of idempotents plays an important rôle. In this note we shall define the kernel of semigroups which have minimal left and minimal right ideals, and investigate the relation between the kernel and minimal left (right) ideals. Thus we propose to extend the theory of bicompact semigroups to more general semigroups.

Let D_1 be a minimal left ideal, D_2 a minimal right ideal, and D the product of D_1 and D_2. Then, in order that a subset $L(R)$ of S be a minimal left (right) ideal of S, it is necessary and sufficient that $L(R)$ be represented in the following form:

$$L=D_1a \qquad (R=aD_2),$$

where a is an element in $L(R)$. From this fact it follows that the product of any minimal left ideal and any minimal right ideal is always equal to D. Therefore D is determined uniquely irrespective of the selection of D_1 and D_2. D is a simple semigroup and is called the kernel of S. If we put $E=D_2D_1$, then E is a group contained in D_1 and D_2. Therefore every semigroup which has its kernel contains at least one idempotent. We obtain the following result, which shows the relation between the kernel and minimal left (right) ideals: the kernel D is decomposed into join of minimal left (right) ideals which have no element in common. On the other hand every minimal left (right) ideal can be divided into groups, which have no element in common. The structure of the kernel D is thus completely determined.

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1. In this paper we limit ourselves to semigroups which have