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Isometric imbedding of Riemann manifolds in a Riemann manifold.

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1. Introduction. S. S. Chern and N. H. Kuiper [11] [1] obtained some theorems concerned with estimates on the lower bound of the dimension of the Euclidean space in which a compact Rieman manifold with some properties can be imbedded isometrically. The object of this paper is to generalize these results to the problem on the isometric imbedding of Riemann manifolds in another Riemann manifold.

The author will also make use of the methods in [11] in certain open sets which S. B. Myers [9] investigated in connection with non-existence of compact minimal subvarieties of dimension $n-1$ in Riemann manifolds of dimension n with some additional properties.

2. μ-domains. Let V_n be a Riemann manifold of dimension $n\geqq 2$ and class C^r [2]. Let O be any point of V_n, let $x^1, x^2, \cdots, x^n$ be geodesic normal coordinates with respect to a rectangular frame (R_0) at O. Let U be a neighborhood of O on which the coordinates are introduced. Let us denote the open set U considering together with the coordinates by $U(O, x)$, put $U=|U(O, x)|$ and call it a *geodesic coordinate neighborhood.* Let us attach to each point $P\in U$ that frame $(R)=\{P, e_i\}, i=1, 2, \cdots, n,$ which we obtain from (R_0) by parallel displacement along the geodesic arc $OP\subset U(O, x)$ [3]. Then, by means of the adapted family of frames [4] to the coordinates, let the connexion of V_n and the structure of the space be given by the following equations

1) Numbers in brackets refer to the list of references at the end of the paper.

2) $r\geqq 4$ is sufficient for all purposes in this paper.

3) By "a geodesic arc $OP\subset U(O, x)$", we shall mean that if $P=(x_0^i)$, the geodesic is given by the equations $x^i=tx_0^i$, $0\leqq t\leqq 1$.

4) See Cartan [1], p. 235.