On some conjectures concerning
pseudo-convex domains.

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The famous problem of E. E. Levi [24], asking whether every pseudo-convex domain is a domain of regularity or not, still awaits a complete solution, though there are a number of deep results concerning this problem.[1] For this, the theory of pluri-subharmonic functions due to P. Lelong ([20]—[23]), K. Oka ([26]), H. J. Bremermann ([10]) and G. O. Thorin ([35]) seems to give a powerful method. This theory allows us in particular to restate the problem of Levi in several forms, in which it may probably be easier to resolve.

In the first and the second chapters of the present paper, we shall give, for later use, a brief sketch of definitions and the main results of the theory of pluri-subharmonic functions and of the domains convex with respect to these functions, respectively. Then, we shall show in the third chapter, that there exist some intimate relations among the conjectures concerning the pseudo-convex domains and the pluri-sub-harmonic functions, as considered by several authors.[2]

For the sake of simplicity, we treat only the domains *univalent* and *finite* (not necessarily *bounded*) in the space $\mathfrak{S}^n$ of n complex variables $(z_1, \cdots, z_n)$. A point or a vector $(z_1, \cdots, z_n)$ in our space $\mathfrak{S}^n$ is sometimes denoted merely by (z_j) or by z.

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1) After completion of this manuscript, the author has been told that Prof. K. Oka solved the problem of Levi affirmatively in its original form. His results will soon appear in his paper [28]. However, it may not be meaningless to consider the relations of this problem to other famous conjectures, and after Oka's results, we obtain some results, seems to be new, as 4d and 4e in Chapter III.

2) For example, Behnke-Stein [5], H. Cartan [13], [14], and Thullen [38].