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A note on unramified abelian covering surfaces of a closed Riemann surface.

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1. Introduction.

Let F be a closed Riemann surface of genus p, and $\tilde{F}$ be an unramified and unbounded covering surface of F. If, above any closed curve on F, there never lie two curves on $\tilde{F}$, one of which is closed and the other open, $\tilde{F}$ is said to be *regular*. As is well known, a regular covering surface $\tilde{F}$ admits *covering transformations* onto itself, which are one-to-one and continuous and carry each point $\tilde{P}$ on $\tilde{F}$ into a point $\tilde{P}'$ with the same projection as $\tilde{P}$. The totality of these transformations forms the *covering transformation group* $\Gamma(\tilde{F})$, which characterizes $\tilde{F}$.

DEFINITION. *A regular covering surface $\tilde{F}$ of F is called an unramified abelian covering surface, if its covering transformation group $\Gamma(\tilde{F})$ is abelian.*[1)]

In the present note, we shall investigate the structure of unramified abelian covering surfaces in some detail (§§ 2–4), and prove some function-theoretic properties of these surfaces (§§ 5–6).

An example of such surfaces is given by the Riemann surface $\tilde{F}_w$ of an *abelian integral* w,[2)] where dw is an analytic differential of the first or the second kind defined on F. $\tilde{F}_w$ is an unramified and unbounded covering surface of F characterized by the following property: a curve $\tilde{\gamma}$ on $\tilde{F}_w$ is closed if and only if its projection γ on F is closed

1) An "unramified abelian covering surface" of a closed Riemann surface corresponds to an "unramified abelian extension" of an algebraic function field. L. Sario [6], [7] used the term "Abelsche Überlagerungsfläche" or "abelian covering surface" for another sort of covering surfaces (one of which is called "die Überlagerungsfläche der Kommutatoren" in [1]), whose covering transformation groups are not abelian except for some simple cases.

2) The Riemann surface of a *multiplicative function* gives a more general example.