Journal of the Mathematical Society of Japan Vol. 6, No. 2, September, 1954

On groups of projective collineations in a space of K-spreads.

By Kentaro YANO and Hitosi HIRAMATU

(Received July 29, 1953)

§0. Introduction.

In an affine space of paths:

$$\frac{d^2x^i}{ds^2}+\Gamma^i_{jk}\frac{dx^j}{ds}\frac{dx^k}{ds}=0, \tag{0.1}$$

$$(a, b, c,\cdots, i, j, k,\cdots=1, 2,\cdots, N)$$

an affine collineation is defined as a point transformation which carries any path into a path and preserves the affine character of the parameter s on it. It was first shown by L. P. Eisenhart and M. S. Knebelman ([5], [6]) that a necessary and sufficient condition that an infinitesimal point transformation

$$\bar{x}^i=x^i+\xi^i(x)dt \tag{0.2}$$

be an affine collineation is that ξ^i satisfy

$$\xi^i{}_{,j,k}+\xi^a\Gamma^i_{jk,a}-\xi^i{}_{,a}\Gamma^a_{jk}+\xi^a{}_{,j}\Gamma^i_{ak}+\xi^a{}_{,k}\Gamma^i_{ja}=0, \tag{0.3}$$

where the comma followed by an index denotes partial differentiation with respect to x^i.

But, if we denote the Lie derivative ([9], [15]) of a geometric object Ω with respect to the infinitesimal point transformation (0.2) by $X\Omega$, then condition (0.3) can be rewritten in a very simple form

$$X\Gamma^i_{jk}\equiv\xi^i{}_{;j;k}+R^i{}_{jkl}\xi^l=0, \tag{0.4}$$

where the semi-colon followed by an index denotes covariant differentiation with respect to Γ^i_{jk}.

It is well known that the maximum order of a group of affine collineations in an N-dimensional space of paths is N^2+N and if the space admits a group of affine collineations of the maximum order, then the space is necessarily affinely flat ([5], [15]).