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On Neumann's problem for a domain on a closed Riemann surface.

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The Neumann's problem is solved usually by means of integral equations. Recently L. Myrberg[1)] proved simply the existence of the solution of the Neumann's problem for the inside of a unit circle, without use of integral equations. By his method, we shall prove the existence of the solution of the Neumann's problem for a domain on a closed Riemann surface, without use of integral equations.

Let F be a closed Riemann surface spread over the z-plane and D be its sub-domain, whose boundary Γ consists of a finite number of analytic Jordan curves or Jordan arcs $\Gamma=\sum_{i=0}^{n}\Gamma_i$, such that, if Γ_i, Γ_{i+1} meet at a point ζ_i, then they make an inner angle $\alpha_i\pi$ $(0<\alpha_i<2)$ at ζ_i. Let $f(\zeta)$ be a given function on Γ, which is continuous on Γ, except at $\{\zeta_i\}$, where $f(\zeta)$ may be discontinuous, but is bounded on Γ, such that

$$|f(\zeta)|\leqq M \qquad \text{on } \Gamma, \tag{1}$$

and satisfies the condition:

$$\int_{\Gamma} f(\zeta)|d\zeta|=0. \tag{2}$$

Then we shall prove

THEOREM. *There exists a harmonic function $u(z)$ in D, which is continuous in $\overline{D}$, such that*

(i) $$|u(z)|\leqq k_1 M \quad \textit{in} \quad \overline{D},$$

where $k_1=k_1(D)$ is a constant, which depends on D only.

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1) L. Myrberg: Über die vermischte Randwertaufgabe der harmonischen Funktionen. Ann. Acad. Sci. Fenn. Series A, 103 (1951).