Journal of the Mathematical Society of Japan Vol. 5, Nos. 3~4, November, 1953.

On the regularity of homeomorphisms of E^n.

By Tatsuo HOMMA and Shin'ichi KINOSHITA

(Received Feb. 16, 1953)

Introduction. Let X be a compact metric space and h a homeomorphism of X onto itself. The homeomorphism h has been called by B. v. Kerékjártó [3][1] *regular* at $p \in X$, if h satisfies the following condition: for each $\epsilon > 0$ there exists $\delta > 0$ such that for each x with $d(p, x) < \delta$ and for each integer m

$$d\big(h^m(p), h^m(x)\big) < \epsilon .$$

One of the purpose of this paper is to prove the following

THEOREM 1. *Let X be a compact metric space and h a homeomorphism of X onto itself. Assume that X and h have the following property: there exist two distinct points a and b such that*

(*i*) *for each point $x \in X - b$ the sequence $\{h^m(x)\}$ converges to a and*

(*ii*) *for each point $x \in X - a$ the sequence $\{h^{-m}(x)\}$ converges to b, where $m = 1, 2, 3, \cdots$.*

Then h is regular at every point of X except for a and b.

As a corollary of Theorem 1 we have the following

THEOREM 2. *Let h be a homeomorphism of the n-dimensional sphere S^n onto itself satisfying the same condition as that of Theorem 1. Then h is regular at every point of S^n except for a and b.*

Now let S^n be the n-dimensional sphere in the $(n+1)$-dimensional Euclidean space E^{n+1} and let P be a point of S^n. Let $p(x)$ be the stereographic projection of $S^n - P$ from P onto the n-dimensional Euclidean space E^n tangent at the antipode O of P, where we assume that O is the origin of E^n. Let h be a homeomorphism of E^n onto itself. Put $\bar{h}(x) = p^{-1}hp(x)$ where $x \in S^n - P$ and put $\bar{h}(P) = P$. Then we have a homeomorphism $\bar{h}$ of S^n onto itself. B. v. Kerékjártó [3] called a

1) The numbers in the brackets refer to the references at the end of this paper.