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On lattice points in an n-dimensional ellipsoid.

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1. Main theorems.

1. Let $\sum_{i,k=1}^{n} a_{ik} x_i x_k$ $(n \geq 2)$ be a positive definite quadratic form with the determinant $D=|a_{ik}|>0$, then

$$\sum_{i,k=1}^{n} a_{ik} x_i x_k \leq r^2 \tag{1}$$

is the inside of an n-dimensional ellipsoid and $V(r)$ be its volume:

$$V(r)=\frac{\pi^{\frac{n}{2}} r^n}{\sqrt{D}\,\Gamma\left(\frac{n}{2}+1\right)}. \tag{2}$$

Let $n(r)$ be the number of lattice points contained in (1) and put

$$n(r)=V(r)+\Omega(r). \tag{3}$$

Then Landau[1] proved that

$$\Omega(r)=O\left(r^{n-\frac{2n}{n+1}}\right) \quad (n \geq 4) \tag{4}$$

and many researches are made concerning the order of $\Omega(r)$ by Landau, Walfisz, Jarnik and others. We shall prove

THEOREM 1. $\int_1^r \frac{\Omega(r)}{r^{n-1}}\,dr=O(1) \qquad (n \geq 2)$.

We remark that the integral diverges, if we put the value of $\Omega(r)$ of (4) in it.

Let $a_i, k_i \geq 0$ $(i=1,2,\cdots,n)$ be integers and consider lattice points $(x_1,\cdots,x_n)$ contained in (1), such that

$$x_i \equiv a_i \pmod{k_i} \quad (i=1,2,\cdots,n) \tag{5}$$

1) E. Landau: Zur analytischen Zahlentheorie der definiten quadratischen Formen. Berliner Akademieber. 1915.