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On ϕ-congruences.

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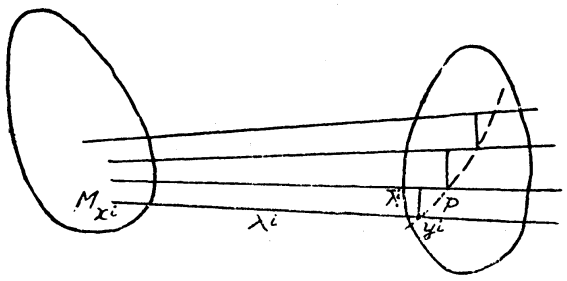
1. Let x^i $(i=1,2,3)$ be the co-ordinates of a point M, on the surface of reference, and λ^i $(i=1,2,3)$ the direction cosines of a line of congruence passing through M. Also, let $\bar{\lambda}^i$ $(i=1,2,3)$ be the direction cosines of a line of another congruence, intersecting the consecutive lines of the given congruence at a constant angle φ. I shall call this the φ-congruence. The line of striction of a ruled surface passing through the original congruence will lie on a surface. This ruled surface will be taken fixed. Ranga Chariar (1945) has shown that the feet of the rays of φ-congruence lie on the line of striction of the given ruled surface. Hence the surface on which this line of striction lies will be taken as the surface of reference of the φ-congruence.

The object of this paper is to find expressions for the parameter of distribution, and the distance of the central point from the surface of reference of the φ-congruence and the equation of ruled surfaces of φ-congruence whose spherical representations are minimal lines. Some particular cases yielding interesting results have been studied.



Surface of reference of the original congruence.

Surface of reference of φ-congruence.

2. Suppose a line of φ-congruence with direction cosines $\bar{\lambda}^i$, $(i=1,2,3)$ intersects its surface of reference at a point P, whose co-ordinates are y^i $(i=1,2,3)$. Then,

$$y^i = x^i + t\lambda^i , \tag{2.1}$$

where t is the distance of the central point of the given ruled surface