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On a direct transcendental singularity of an inverse function of a meromorphic function.

By Masatsugu TSUJI

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Let Δ be an infinite domain on the z-plane, which may be infinitely multiply connected and Γ be its boundary, which consists of at most a countable number of analytic curves. We assume that Γ contains at least one curve extending to infinity. Let $w=w(z)$ be regular in Δ and on Γ, except at $z=\infty$, such that $|w(z)|<R$ in Δ and $|w(z)|=R$ on Γ and $w(z)\neq 0$ in Δ. Let Δ_r be the part of Δ, which lies in $|z|<r$. We put

$$S(r;\Delta)=\frac{1}{\pi}\iint_{\Delta_r}\frac{|w'|^2}{(1+|w|^2)^2}dxdy,\ (w=w(z),\ z=x+iy), \tag{1}$$

$$T(r;\Delta)=\int_1^r\frac{S(r;\Delta)}{r}dr. \tag{2}$$

Now Δ_r consists of a finite number of connected domains. Let Δ_r^0 $(r\geqq r_0)$ be the one, which contains a fixed point z_0 of Δ and θ_r be the part of $|z|=r$, which belongs to the boundary of Δ_r^0. θ_r consists of a finite number of arcs θ_r^i $(i=1,2,\cdots,\nu(r))$ and $r\theta_i(r)$ be its arc length and put $\theta(r)=\sum\theta_i(r)$. $\theta(r)$ is continuous except at most a countable number of isolated points $0<r_1<r_2<\cdots<r_\nu\to\infty$, where $\theta(r_\nu-0)=\theta(r_\nu)<\theta(r_\nu+0)$.

In the former paper,[1] I have proved the following theorem.

THEOREM. *For any* $0<\alpha<1$,

$$T(r;\Delta)\geqq\text{const. } e^{\pi\int_{r_0}^{\alpha r}\frac{dr}{r\theta(r)}}\ (r\geqq r_0).$$

1) M. Tsuji: On a regular function which is of constant absolute value on the boundary of an infinite domain. Tohoku Math. Journ. 3 (1951).