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Generalized l^p spaces and the Schur property.

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1.—The following situation (essentially) was considered by H. Nakano [3]. This problem was considered by W. Orlicz [5] in a restricted form. Let J be a collection, not necessarily countable, of marks α. For given J-sequences $p=\{p(\alpha)\}$, $w=\{w(\alpha)\}$ with $p(\alpha)\geqq 1$ and $w(\alpha)>0$ for all α, let $l=l(p, w)$ denote the space of all real or complex valued J-sequences $x=\{x(\alpha)\}$ for which $\|x\|$ is finite; here, by definition,

(1.1) $\|x\|=\inf\eta$ for all $\eta>0$ with $\sum w(\alpha)\left|\frac{x(\alpha)}{\eta}\right|^{p(\alpha)}\leqq 1$ the symbol $\sum$ indicating that the non-zero addends are denumerable and have an absolutely convergent sum in the usual sense (if there are no such η then $\|x\|$ is defined to be ∞). The notation $l(p, w)$ may be replaced by $l(p)$ if $w(\alpha)=1$ for all α, and by l_p if, in addition, $p(\alpha)=p$ (a constant) for all α.

If R, S are two collections of J-sequences, $R\cong S$ shall mean that numbers $m(\alpha)$ exist such that the relations $y(\alpha)=m(\alpha)\,x(\alpha)$ set up a (1,1) correspondence between all x in R and all y in S.

A Banach space is said to have the Schur property if every weakly convergent sequence of its elements is necessarily convergent in norm (as shown by J. Schur [4], l^1, with J the set of all positive integers, has this property).

2.—The arguments used in [3] show:

(I): Every $l(p, w)$ is a Banach (i. e., linear, normed and complete) space.

(II): $l(p, w_1)\cong l(q, w_2)$ if and only if

(2.1) $\sum\theta^{\frac{p(\alpha)\,q(\alpha)}{[p(\alpha)-q(\alpha)]}}<\infty$ for some $0<\theta<1$, the sum to be taken over all α for which $p(\alpha)\neq q(\alpha)$.

(III): $l(p, w)$ has the Schur property if

(2.2) for every $\varepsilon>0$ the α for which $p(\alpha)>1+\varepsilon$ are finite in number.

(IV): There are $l(p, w)$ with the Schur property for which $l(p, w)\cong l^1$ is false.