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Principal ruled surfaces of a rectilinear congruence.

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Introduction.

Let p^{ij} be the Plücker coordinates of a line p in projective three dimensional space $\boldsymbol{R}_3$. If p $(p^{01}, p^{02}, p^{03}, p^{12}, p^{13}, p^{23})$ is a function of two parameters u^1 and u^2, the line p describes a rectilinear congruence K when u^1 and u^2 vary. Now put[1)]

$$p^i=\frac{\partial p}{\partial u^i}(i=1,2),\ \ -((p_i\,p_j))=H_{ij}\ \ (i,j=1,2).$$

If the determinant determined by the elements H_{ij} $(i,j=1,2)$ does not vanish identically, the congruence K has two focal surfaces S_0 and S_1. We restrict ourselves in this case.

Let us consider the image of a line p in the projective five dimensional space $\boldsymbol{R}_5$, the plane $\boldsymbol{S}_2$ determined by the three points p, p_1 and p_2 is the tangential plane of the image V of K at p, and the plane $\boldsymbol{S}_4$ determined by $\boldsymbol{S}_2$ and its conjugate $\boldsymbol{S}_2'$ with respect to the quadric of Plücker $\boldsymbol{Q}_4$ is the polar plane of $\boldsymbol{Q}_4$ at p, that is, the tangential plane of $\boldsymbol{Q}_4$ at p. Let p_5 be a point which does not lie in this tangential hyperplane $\boldsymbol{S}_4$, the plane $pp_1p_2p_5$ has no common point with the conjugate $\boldsymbol{S}_1'$ with respect to $\boldsymbol{Q}_4$, $\boldsymbol{S}_1'$ intersects with $\boldsymbol{Q}_4$ at two different points p_3 and p_4. Then p_3 and p_4 lie on the tangential hyperplane $pp_1p_2p_3p_4$, and the lines pp_k, p_kp_5 $(k=3,\ 4)$ are not conjugate to each other. Moreover, to determine uniquely the point p_5, we select p_5 as the intersection of $\boldsymbol{Q}_4$ and the line joining the point p and $\dfrac{1}{2}H^{\sigma\tau}\dfrac{\bar{\partial}^2 p}{\partial u^\sigma \partial u^\tau}$ ($\bar{d}$ denotes absolute differentiation). Then we have the fundamental equations for the given congruence K as follows[2)]: