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Theory of invariants in the geometry of paths.

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Introduction.

In an n-dimensional space X_n referred to a coordinate system x^i $(i=1, 2, \cdots, n)$, we consider a system of ordinary differential equations of the m-th order

$$x^{(m)i}+H^i(t, x, x^{(1)}, \cdots, x^{(m-1)})=0 \tag{0.1}$$

where $x^{(r)i}=d^r x^i/dt^r$ $(r=1, 2, \cdots, m)$. Its solutions $x^i=x^i(t)$ which exist under suitable conditions for the functions H^i determine a system of curves called *paths of the m-th order.* We assume in the following that the functions H^i admit continuous derivatives with respect to the $mn+1$ arguments t, x^i, $x^{(1)i}, \cdots, x^{(m-1)i}$ up to the order needed.

Hitherto many has been contributed to the theory of invariants of the paths under various transformation groups, by which various geometries of paths were established. First of all, we notice the result of A. Kawaguchi and H. Hombu [8][1]. This is concerned with the theory under the transformation group of coordinates

$$\xi^\alpha=\xi^\alpha(x^i), \quad \tau=t, \tag{i}$$

and also with the theory under the transformation group of coordinates and parameter

$$\xi^\alpha=\xi^\alpha(x^i), \quad \tau=\tau(t). \tag{ii}$$

The first case was also treated by D. D. Kosambi [7]. We call this *the ordinary geometry of paths.* Later the second case was treated by T. Ohkubo [9] and S. Hokari [4] in the case of the third and m-th order respectively. These studies arrived at remarkable results. We call this type of geometry *intrinsic.*

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1) Numbers in brackets refer to the bibliography at the end of the paper.