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On the change of variables in the multiple integrals.

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The well-known formula on the change of variables in the multiple integrals

$$(*)\quad \int_{f(D)} g(y)dy=\int_D g(f(x))\mathrm{abs}\left|\frac{\partial f}{\partial x}\right|dx$$

has been proved by H. Rademacher and M. Tsuji under very general assumptions. They have shown that the functions f satisfying certain conditions are totally differentiable almost everywhere and consequently the Jacobian $\left|\frac{\partial f}{\partial x}\right|$ can be defined almost everywhere. They have proved further that the above formula $(*)$ holds for integrable functions g, and f satisfying these conditions. We shall give in the following lines another proof of the last fact. Namely we suppose f as a. e. totally differentiable, g as integrable and show the validity of $(*)$. (For the exact formulation see below.) We treat further the case where f is not necessarily univalent.

Throughout this paper, we shall concern ourselves with subsets and mappings of the euclidean n-space E^n. f represents always a mapping defined on a certain subset of E^n. Letters like x, y, a, b represent points of E^n. $\|x-y\|$ denotes the distance between x and y.

§1. Preliminaries.

DEFINITION 1. *A mapping $f(x)$ defined on a bounded domain $D(\subset E^n)$ is called an $\mathfrak{A}$-function on D, if it satisfies the following three conditions.*

($\mathfrak{A}_1$) *f maps D homeomorphically onto $f(D)$.*

($\mathfrak{A}_2$) *If $\mu(E)=0$ $(E\subset D)$, then $\mu(f(E))=0$.*

($\mathfrak{A}_3$) *$f(x)$ is totally differentiable almost everywhere.*