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On uniform topologies in general spaces.

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The important notion of the uniformity, introduced by A. Weil[1] and others, shows its full effectiveness, when the space is completely regular. However, we can define the "generalized uniformity" of any neighbourhood space, as an arbitrary collection of correspondences assigning to every point of the space a neighbourhood. We shall show in § 2 of this paper, that the most part of the theory of uniformity holds also in the spaces with the generalized uniformity. We can consider also the completion of such spaces in several manners (§§ 3, 4). The usual way of completion by means of Cauchy filters (we have named it C-extension, § 5) does not give a complete space in general cases. In § 6 we shall consider some additional conditions on such spaces, and investigate the behaviour of the C-extensions of spaces satisfying these conditions.

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§ 1 Generalized uniformity.

1.1 Definition. We say that X is a *space*, if X is an aggregate of points, where a closure operator is defined which assigns to each subset M of X a closure $\overline{M}$ with the following properties:

(1) $\overline{M}\supset M$, (2) $M_1\subset M_2$ implies $\overline{M}_1\subset \overline{M}_2$, (3) $\overline{\phi}=\phi$.

Topological concepts, such as *neighbourhood* (abbr. nbd) of a point, continuity of mappings, etc., may be defined in our space in the well-known way.[2]

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1) A. Weil: Sur les espaces à structure uniforme et sur la topologie générale. Actual. Sci. Ind. 551 (1938).

2) cf. e. g. J. W. Tukey: Convergence and uniformity in topology. Princeton Univ. (1940).