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On the Schur relations for the representations of a Frobenius algebra.

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The Schur relations for the representations of a Frobenius algebra was studied in [1], [6][1]. In the present note we shall prove the Schur relations by a new method. Some supplementary results are also obtained. In § 1 we shall study the properties of corresponding bases[2] of a Frobenius algebra. § 2 deals with the Cartan basis[3] of an algebra. Using the results obtained in §§ 1 and 2, we shall derive in § 3 the Schur relations for the representations of a Frobenius algebra.

1. Corresponding bases of a Frobenius algebra. We consider an algebra A with unit element over a given field K. Let $u_1, u_2, \cdots, u_n$ be a basis of A. Let us denote by $S(a)$ and $R(a)$ the left and the right regular representations of A defined by the basis (u_i):

$$a(u_i)\,b=(u_i)\,S(a)\,R'(b) \qquad (a,\ b\ in\ A) \tag{1}$$

where $R'(b)$ is the transpose of $R(b)$. A is called a Frobenius algebra if $S(a)$ is similar to $R(a)$:

$$S(a)=P^{-1}\,R(a)\,P\,. \tag{2}$$

We then have

$$(P')^{-1}\,R(a)\,P'=S(a^{\varphi}) \qquad (a^{\varphi}\ in\ A)\,. \tag{3}$$

The mapping $a\rightarrow a^{\varphi}$ forms an automorphism φ of A. This automorphism is completely determined by A, apart from an inner automorphism. We see that

$$(u_i^{\varphi})=(u_i)\,(P')^{-1}\,P \tag{4}$$

where (u_i^{φ}) is obtained from (u_i) by application of the automorphism $\varphi: a\rightarrow a^{\varphi}$. If we set

1) The numbers in the brackets refer to the references at the end of the paper.

2) Brauer [1].

3) Nesbitt [4], Nesbitt and Scott [5].