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Some Remarks on the Theory of Picard Varieties

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Let $\boldsymbol{V}^d$ be a non-singular projective model in the algebraic geometry with the universal domain of all complex numbers and let

$$(\gamma)=(\gamma_1, \gamma_2, \dots, \gamma_{2q})$$

be a base of the first integral Betti group of $\boldsymbol{V}$, then we can find the "invariant cycles" on the generic curve[1] $\boldsymbol{C}(\boldsymbol{M})$ in $\boldsymbol{V}$

$$(\beta)=(\beta_1, \beta_2, \dots, \beta_{2q}),$$

which is homologous to (γ) modulo Q. Moreover let ω be the period matrix of the Picard integrals of the first kind in $\boldsymbol{V}$ along (γ) and let

$$E={}^t I_\beta^{-1}$$

be the transposed inverse of the intersection matrix of the invariant cycles on $\boldsymbol{C}(\boldsymbol{M})$ then E is one of the principal matrices of the Riemann matrix ω.

We have also attached the *Albanese variety* $\boldsymbol{A}^q$ and the *Picard variety* $\boldsymbol{P}^q$ to the Riemann matrices ω and

$$\hat{\omega}=\omega\epsilon^{-1}E,$$

where ϵ means the Pfaffian of E. More precisely if we denote by $[\omega]$ the discrete subgroup of rank $2q$ in the complex vector space S^q, then $\boldsymbol{A}^q$ is isomorphic with the complex toroid $S^q/[\omega]$; and similary for $\boldsymbol{P}^q$ and $\hat{\omega}$.

On the other hand let

$$(\Gamma)=(\Gamma_1\ \Gamma_2, \dots, \Gamma_{2q})$$

be a base of the integral homology group of $(2d-1)$ dimension in $\boldsymbol{V}$, which is dual to (γ) in the sense

$$I(\gamma_i, \Gamma_j;\ \boldsymbol{V})=\delta_{ij} \qquad (1\leq i, j\leq 2q).$$

Then if we put

$$\hat{\gamma}_i=\Gamma_i\cdot\boldsymbol{C}(\boldsymbol{M}) \qquad (1\leq i\leq 2q),$$

We shall use freely the results and terminology of Weil's book: Foundations of algebraic geometry, Am. Math. Soc. Colloq., Vol. 29 (1946).

1) See my paper, On the Picard varieties attached to algebraic varieties, Amer. J. of Math. Vol. 74 (1952). We cite this paper as (P).