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Factor System Approach to the Isomorphism and Reciprocity Theorems

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The Takagi-Artin class field theory establishes a canonical isomorphism between the Galois group of the full Abelian extension A_k of an algebraic number field k and a certain factor group of the idèle-class group $\mathfrak{C}_k$ of k, according to Chevalley's formulation ([2], [3]). Let K be a (finite) Galois extension of k and let A_K be the full Abelian extension of K. Then the Galois group of A_K over k is an extension of the Galois group of A_K/K by the Galois group $\mathfrak{G}$ of K/k. Thus it defines, by the cited canonical isomarphism, a factor system of $\mathfrak{G}$ in a factor group of the idèle-class group $\mathfrak{C}_K$ of K. Weil showed recently that this factor system can be represented by a factor system in the idèle-class group itself, so as some further requirements are met ([12]). Now, it is hoped to construct such factor systems directly and to use them conversely in establishing the class field theory. For the lodal class field theory such a factory system approach has been given in the note [8].[1] As to the global theory, i.e. the class field theory proper, Hasse has shown that his sum-relation of locol invariants of Brauer algebra-classes gives a central assertion in the reciprocity law ([5]). Noether has given a factor system formulation of the principal genus theorem ([10]). To proceed further (or, to start with, rather), it is desirable to define certain canonical factor systems in idèle-class groups and to derive from them the isomorphism and reciprocity theorems directly. This we propose to do in the present note.[2] Thus the work has little novelty in its true arithmetical bearing. But it provides, as the writer hopes, a rather elegant approach to those thorems.

1) Cf. also [9] and Akizuki [1]. Further, Hochschild [6] has constructed the whole theory without appealing to the theory of algebras but by dealing merely with factor systems (or cohomology classes).

2) Hochschild [7] has given recently a very direct and elegant proof to the reciprocity law (using also the idea of Hasse but without dealing with algebras explicitely). However, while he combines local canonical isomorphisms defined by local factor systems, in order to obtain global canonical isomorphism, so to speak, our present program is to carry out the transition to "global" already at the stage of factor systems, which also clarifies the combination of isomorphisms.