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On the weak Topology of an infinite Product Space.

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1. Introduction. We shall define a monotonic topology of a space R as a closure operator which assigns to each subset M of R a closure $\bar{M} \subset R$ with following properties

$$\bar{0}=0, \qquad M \supset N \rightarrow \bar{M} \supset \bar{N}.$$

If we assume furthermore

$$\overline{M \cup N} \subset \bar{M} \cup \bar{N},$$

then we say that the topology is additive.

In this note we define a weak monotonic topology and from it a weak additive topology of an infinite product space by means of the closure operator, and show that these topologies are the weakest respectively in all allowable topologies.

2. Let $R=P\{R^x \mid X\}$ be the $X=\{x\}$ product space of R^x whose points are $p=\{p^x \mid p^x \in R^x,\ x \in X\}$. Usually the topology of R is necessarily to satisfy the condition that the projection $\pi^x : R \rightarrow R^x$ is continuous. This condition is expressed by the closure operator as follows:

$$\pi^x(\bar{M}) \subset \overline{\pi^x(M)} = \overline{M^x} \quad \text{for any} \quad M \subset R, \tag{1}$$

where the left side closure means that in R, and the right side closure in R^x.

If we define

$$^{m}\bar{M}=P\{\overline{M^x} \mid x \in X\} \quad \text{for any} \quad M \subset R,$$

this closure determines a monotonic topology of R, for it follows that

$$M \supset N \rightarrow M^x \supset N^x \rightarrow \overline{M^x} \supset \overline{N^x} \rightarrow P\{\overline{M^x} \mid X\} \supset P\{\overline{N^x} \mid X\}.$$

Clearly this topology $^{m}\bar{M}$ is the weakest in all topologies of R satisfying (1).

3. We shall define now the weakest additive topology of R. Let μ be a finite subdivision of $M(\subset R)$,

$$\mu : M=M_1 \cup \cdots\cdots \cup M_{n(\mu)},$$