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On Macaulayfication of certain quasi-projective schemes

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1. Introduction.

Let X be a Noetherian scheme. A birational proper morphism $Y \to X$ of schemes is said to be a *Macaulayfication* of X if Y is a Cohen-Macaulay scheme. This notion was introduced by Faltings [**8**] and he established that there exists a Macaulayfication of a quasi-projective scheme over a Noetherian ring possessing a dualizing complex if its non-Cohen-Macaulay locus is of dimension 0 or 1. Of course, a desingularization is a Macaulayfication and Hironaka gave a desingularization of arbitrary algebraic variety over a field of characteristic 0. But Faltings' method to construct a Macaulayfication is independent of the characteristic of a scheme. Furthermore, several authors are interested in a Macaulayfication.

For example, Goto and Schenzel independently showed the converse of Faltings' result in a sense. Let A be a Noetherian local ring possessing a dualizing complex, hence its non-Cohen-Macaulay locus is closed, and assume that $\dim A/\mathfrak{p} = \dim A$ for any associated prime ideal $\mathfrak{p}$ of A. Then the non-Cohen-Macaulay locus of A consists of only the maximal ideal if and only if A is a generalized Cohen-Macaulay ring but not a Cohen-Macaulay ring [**16**]. When this is the case, Faltings [**8**, Satz 2] showed that there exists a parameter ideal $\mathfrak{q}$ of A such that the blowing-up $\operatorname{Proj} A[\mathfrak{q}t]$ of $\operatorname{Spec} A$ with center $\mathfrak{q}$ is Cohen-Macaulay, where t denotes an indeterminate. Conversely, Goto [**9**] proved that if there is a parameter ideal $\mathfrak{q}$ of A such that $\operatorname{Proj} A[\mathfrak{q}t]$ is Cohen-Macaulay, then A is a generalized Cohen-Macaulay ring. Moreover, he showed that A is Buchsbaum if and only if $\operatorname{Proj} A[\mathfrak{q}t]$ is Cohen-Macaulay for every parameter ideal $\mathfrak{q}$ of A: see also [**20**].

Brodmann [**3**] also studied the blowing-up of a generalized Cohen-Macaulay ring with center a parameter ideal. Furthermore, he constructed Macaulayfications in a quite different way from Faltings. Let A be a Noetherian local ring possessing a dualizing complex. We let $d = \dim A$ and s be the dimension of its non-Cohen-Macaulay locus. If $s = 0$, then Brodmann [**4**, Proposition 2.13] gave an ideal $\mathfrak{b}$ of height $d-1$ such that $\operatorname{Proj} A[\mathfrak{b}t]$ is Cohen-Macaulay. If $s = 1$, then Faltings' Macaulayfication [**8**, Satz 3] of $\operatorname{Spec} A$ consists of two consecutive blowing-ups $Y \to X \to \operatorname{Spec} A$ where the center of the first blowing-up is an ideal of height $d-1$. In this case, Brodmann gave two other Macaulayfications of $\operatorname{Spec} A$: the first one [**1**] is the composite of a blowing-up $X \to \operatorname{Spec} A$ with center an ideal of height $d-1$ and a finite morphism $Y \to X$; the second one [**4**, Corollary 3.11] consists of two consecutive blowing-ups $Y \to X \to \operatorname{Spec} A$ where the center of the first blowing-up is an ideal of height $d-2$.

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