J. Math. Soc. Japan
Vol. 50, No. 4, 1998

On the structure of the singular set of a complex analytic foliation

By Junya YOSHIZAKI

(Received Oct. 5, 1995)
(Revised Nov. 20, 1996)

0. Introduction.

A singular foliation on a complex manifold M is determined by an involutive coherent subsheaf E of the tangent sheaf of M. In this paper, we study the problem of local (analytical and topological) triviality of the singular foliation along (a subset of) its singular set $S(E)$. In general, $S(E)$ is an analytic variety, so we examine by stratifying the set.

For stratified subsets or stratified maps, the local topological triviality has been studied by a number of people and it is generally known that if the stratification satisfies the "Whitney condition" or the "Thom condition", then we have the local topological triviality along each stratum (the Isotopy Lemmas of Thom).

We first consider the case of analytical triviality in section 2 of this paper, after reviewing basic definitions and facts on complex analytic singular foliations in section 1. For complex analytic singular foliations we have the fundamental "Tangency Lemma" (Theorem (2.5)), which says that every vector field defining the foliation is "tangential" to the singular set $S(E)$. We discuss and summarize the implications of this lemma, which include the existence of the integral submanifold (leaf) through each point of M (even on $S(E)$) and the local analytical triviality of the foliation along each leaf.

As another application of the Tangency Lemma, we prove, for a complex analytic singular foliation E, the existence of a Whitney stratification of the singular set $S(E)$ so that E induces a non-singular foliation on each of its strata (Theorem (3.4)).

In section 4, we study the local topological triviality along each stratum of a stratification of $S(E)$ as given in Theorem (3.4). This kind of triviality argument can be applied to the case where a stratum consists of (infinitely) many leaves. A. Kabila studied this problem for the case where the codimension of E is one and $S(E)$ is non-singular ([**K**]). We give, for a general singular foliation, a regularity condition and prove the local topological triviality under the condition (Theorem (4.10)).

After the preparation of the manuscript, it was informed to me that a result similar to the last one is indicated in an article of D. Trotman and L. Wilson [**TW**].

In the process of this work, I received many helpful suggestions and advices, especially from T. Suwa. I would like to thank him for answering my questions and for supporting me in various ways. I also thank J.-P. Brasselet, M. Kwieciński, T. Ohmoto, A. Saeki and the referee for helpful conversations and comments.

1. Complex analytic singular foliations.

First of all, we recall some general facts about singular foliations on complex manifolds and fix the notations used in this papar. For further details, see [**BB**] and [**Sw**].