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On the Besov-Hankel spaces*

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1. Introduction and preliminaries.

Consider the Hankel transformation h_μ defined for suitable functions ϕ by

$$h_\mu(\phi)(x) = \int_0^\infty y^{2\mu+1}(xy)^{-\mu}J_\mu(xy)\phi(y)\,dy, \quad x \in (0,\infty),$$

where J_μ represents the Bessel function of the first kind and order μ. Here and in the sequel μ is a real number greater than $-1/2$. The convolution for the transformation h_μ is defined through

$$(\phi \# \psi)(x) = \int_0^\infty (\tau_x\phi)(y)\psi(y)\,d\gamma(y), \quad x \in (0,\infty),$$

where the Hankel translation operator $\tau_x, x \in (0,\infty)$, is given by

$$(\tau_x\phi)(y) = \int_0^\infty D(x,y,z)\phi(z)\,d\gamma(z), \quad x,y \in (0,\infty),$$

being $d\gamma(x) = (x^{2\mu+1}/2^\mu\Gamma(\mu+1))\,dx$ and

$$D(x,y,z) = \frac{2^{3\mu-1}\Gamma(\mu+1)^2}{\Gamma(\mu+1/2)\sqrt{\pi}}(xyz)^{-2\mu}A(x,y,z)^{2\mu-1}, \quad x,y,z \in (0,\infty).$$

Here $A(x,y,z)$ is the area of a triangle with sides x,y,z when such a triangle exists and $A(x,y,z) = 0$ otherwise.

In earlier papers ([**6**] and [**9**]) the $\#$-convolution have been investigated on the spaces L_μ^p defined for $1 \le p < \infty$ to consist of those complex-valued functions ϕ, measurable on $(0,\infty)$ and such that $\|\phi\|_{p,\mu} < \infty$, where

$$\|\phi\|_{p,\mu} = \left\{\int_0^\infty |\phi(x)|^p x^{2\mu+1}\,dx\right\}^{1/p}.$$

By L^∞ we denote as usual the space of essentially bounded measurable functions on $(0,\infty)$ and $\|\ \|_\infty$ represents the usual norm in L^∞. The space of compactly supported continuous functions on $(0,\infty)$ is denoted by C_0.

Let $T \in (0,\infty)$. We define the Bochner-Riesz mean $\sigma_T^\beta(\phi)$ of a measurable function ϕ on $(0,\infty)$ by

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